ATTENUATION AND POWER-HANDLING CAPABILITY OF HELICAL RADIO-FREQUENCY LINES

J.H. Bryant and E.J. White
Federal Telecommunication Laboratories, a division of
International Telephone and Telegraph Corporation
Nutley, New Jersey

Abstract

A method is described for computing cold insertion loss as well as power-handling capabilities of a helical radio-frequency line. The helically-conducting-cylinder model in free space is considered first. Computations are then extended to account for the presence of an outer, coaxial, uniformly conducting cylinder.

Attenuation

An expression for the loss due to imperfect conductivity will be derived by applying the helically-conducting-sheet model, which is shown surrounded by a coaxial cylinder in Figure 1, to a helix. The power flow along the helix is given by

$$P_T = P_0 \exp(-2\alpha z),$$

where P_T is the transmitted power, P_0 is the initial input power, α is the attenuation in nepers per unit length, and z is the axial distance from the input of the helix. This can be expressed as

$$\alpha = -\frac{1}{2P_T}\frac{dP_T}{dz} \tag{1}$$

The quantity P_T is given by Pierce[1] as

$$P_T = B^2 \frac{k\beta}{\gamma^4}\frac{1}{F^3(\gamma a)} \tag{2}$$

The only factor remaining to be evaluated is dP_T/dz, which may be obtained by considering a cylindrical element of the helically conducting sheet. The area of such an element of length dz is $2\pi a\, dz$, where a is the radius of the cylinder. If P_L is the power loss per unit area, then the total loss over the element is

$$dP_T = 2\pi a\, P_L\, dz \tag{3}$$

The power loss per unit area P_L is found by using the customary method of computing attenuation of radio-frequency lines as follows. If E and H are the electric and magnetic fields, respectively, n is a unit vector normal to the surface of the sheet, and R_s is the skin-effect resistance, $E = R_s(n \times H)$. Hence, $E_z = R_s H_\theta$; $E_\theta = -R_s H_z$. The power flow, given by the Poynting vector, becomes

$$P_L \equiv 1/2\ (E \times H) = (1/2)\ R_s\ (|H_\theta|^2 + |H_z|^2) \tag{4}$$

where R_s is the skin-effect resistance. The skin-effect resistance is given by $R_s = (\pi \mu_0 f \rho)^{1/2}$ where μ_0 is the permeability of free space and ρ is the resistivity of the helix material (see Figure 2 for specific values). The field solutions from Pierce[2] for the helically conducting sheet in free space are used to evaluate (4) by analysis of the fields both inside and outside the helix structure. Inside the helix

$$H_\theta = j\ \frac{B}{(\mu/\epsilon)^{1/2}}\ \frac{k}{\gamma}\ I_1(\gamma r)\ ,$$

$$H_z = -j\ \frac{B}{(\mu/\epsilon)^{1/2}}\ \frac{\gamma}{k}\ \frac{I_0(\gamma a)}{I_1(\gamma a)}\ \frac{1}{\cot \psi}\ I_0(\gamma r)\ .$$

Outside the helix

$$H_\theta = -j\ \frac{B}{(\mu/\epsilon)^{1/2}}\ \frac{k}{\gamma}\ \frac{I_0(\gamma a)}{K_0(\gamma a)}\ K_1(\gamma r)\ ,$$

$$H_z = j\ \frac{B}{(\mu/\epsilon)^{1/2}}\ \frac{\gamma}{k}\ \frac{I_0(\gamma a)}{K_1(\gamma a)}\ \frac{1}{\cot \psi}\ K_0(\gamma r)\ .$$

The total loss per unit area, the sum of the inside and outside losses, becomes

$$P_L = \frac{B^2 R_s}{2.88 \times 10^4 \pi^2}\ (I_1^2 K_0^2 + I_0^2 K_1^2) \left[\left(\frac{k}{\gamma}\right)^2 \frac{1}{K_0^2}\ \frac{1}{\cot^2 \psi}\ \left(\frac{\gamma}{k}\right)^2\ \frac{I_0^2}{I_1^2 K_1^2} \right], \tag{5}$$

where the argument of all the Bessel functions is γa.

Solving for α, we obtain

$$\alpha = -\frac{\pi}{a}\ \frac{(\gamma a)^4 F^3(\gamma a)}{B^2\ (ka)(\beta a)}\ P_L\ . \tag{6}$$

Combining (5) and (6)

$$\frac{\alpha a}{R_s} = \frac{(\gamma a)^4}{2\pi (120)^2}\ \frac{F^3(\gamma a)}{(ka)(\beta a)}\ \left[I_1^2(\gamma a)\ K_0^2(\gamma a) + I_0^2(\gamma a)\ K_1^2(\gamma a) \right]$$

$$\times \left[\left(\frac{k}{\gamma}\right)^2 \frac{1}{K_0^2(\gamma a)} + \frac{\gamma}{k}\ \frac{1}{\cot^2 \psi}\ \frac{I_0^2(\gamma a)}{I_1^2(\gamma a)\ K_1^2(\gamma a)} \right]. \tag{7}$$

In Figure 2 are plotted the graphs of $\alpha a/R_s$ versus γa, with $\cot \psi$ as parameter.

Power Handling

With a satisfactory method of computing the attenuation of a helix having been given, the remaining problem of power-handling capabilities will be solved.

For a helix structure operating as a radio-frequency transmission line in a vacuum, where conduction losses are insignificant, the initial input power P_0 is

$$P_0 = P_T + P_R \ , \tag{8}$$

where P_T is the total transmitted power and P_R is the total power radiated from the structure.

The total power radiated from a helix structure can be written as

$$P_R = 2\pi azW \ , \tag{9}$$

where a is the helix radius, z is the axial length, and W is the power radiated per unit area, which is a function of temperature and the helix material as well as of surface conditions. From (1) and Figure 2, it can be shown that for any given helix structure

$$\alpha = \frac{kR_s}{a} \ , \tag{10}$$

where k is a function of γa and cot ψ. Substituting (10) in (1), the total transmitted power is

$$P_T = P_0 \exp\left(-2kR_s z/a\right) \ , \tag{11}$$

and substituting (9) and (11) in (8), we obtain

$$P_0 = \frac{2\pi azW}{1-\exp(-2kR_s z/a)} \ . \tag{12A}$$

Now assuming that $2kR_s z/a$ is small compared to unity as is true for all structures considered here, the input power P_0 can be expressed as

$$P_0 = \frac{\pi a^2 W}{kR_s} \tag{12B}$$

The power radiated per unit area W is given by

$$W = e_t\, \sigma\, (T^4 - T_0^4), \text{ watts per square centimeter,} \tag{13}$$

where e_t is the total emissivity of the helix material, σ is the Stefan-Boltzmann constant, T is the helix temperature in degrees Kelvin, and T_0 is the ambient temperature.

A graph of $P_0R_s/a^2e_t\,\sigma\,(T^4-T_0^4)$ versus γa is plotted for various values of cot ψ in Figure 3.

Effect of Outer, Coaxial, Conducting Cylinder

In practice, the occasion may arise for the use of a helix inside a coaxial, uniformly conducting cylinder as a radio-frequency transmission line (see Figure 1). Since the presence of the outer cylinder will modify the field components with respect to those of a helix in free space, it is to be expected that the degree of attenuation and the power-handling capability would also be altered. Expressions for the field components are given in Reference 3. When these are substituted into (4), the attenuation factor, corresponding to (7), for the case where the helically conducting sheet and surrounding cylinder are of the same material and at the same temperature, becomes

$$\frac{\alpha a}{R_s} = \frac{(\gamma a)^4 F^3(\gamma a, \gamma b)}{2\pi (120)^2 \, (ka)(\beta a)} \left\{ I_0^2(\gamma a) \left(\frac{k}{\gamma}\right)^2 \left\{ \frac{I_1^2(\gamma a)}{I_0^2(\gamma a)} + \left[\frac{I_1(\gamma a)\, K_0(\gamma b) + K_1(\gamma a)\, I_0(\gamma b)}{I_0(\gamma a)\, K_0(\gamma b) - K_0(\gamma a)\, I_0(\gamma b)}\right]^2 + \left[\frac{I_1(\gamma b)\, K_0(\gamma b) + I_0(\gamma b)\, K_1(\gamma b)}{I_0(\gamma a)\, K_0(\gamma b) - K_0(\gamma a)\, I_0(\gamma b)}\right]^2 \right\} + \left(\frac{\gamma}{k}\right)^2 \frac{I_0^2(\gamma a)}{\cot^2 \psi} \left\{ \frac{I_0^2(\gamma a)}{I_1^2(\gamma a)} + \left[\frac{I_0(\gamma a)\, K_1(\gamma b) + K_0(\gamma a)\, I_1(\gamma b)}{I_1(\gamma a)\, K_1(\gamma b) - K_1(\gamma a)\, I_1(\gamma b)}\right]^2 + \left[\frac{I_1(\gamma b)\, K_0(\gamma b) + I_0(\gamma b)\, K_1(\gamma b)}{I_1(\gamma a)\, K_1(\gamma b) - K_1(\gamma a)\, I_1(\gamma b)}\right]^2 \right\} \right\} \qquad (14)$$

where b is the inside radius of the cylinder. A graph of this function for a ratio of cylinder-to-helix radius b/a equal to 2 is shown in Figure 4. A corresponding graph for power handling is shown in Figure 5.

By comparing Figure 3 with Figure 6, it may be noted that the effect of the outer, uniformly conducting cylinder of diameter twice the helix diameter seems to be to increase the power-handling capability slightly.

The above computations of power-handling capability were made for a helix without dielectric supports. This approximation has some justification, since although dielectric support material may give added loss, the dielectric supports will give more surface area for heat radiation.

Conclusions

A method has been described for calculating the cold insertion loss as well as the power-handling capabilities of a helix structure with or without an outer, uniformly conducting, coaxial cylinder.

References

1. J. R. Pierce, Traveling-Wave Tubes, D. Van Nostrand Company, New York, New York; 1950: Appendix 2.

2. Page 231 of reference 1.

3. J. H. Bryant, "Wave Properties of Helical Conductors," to be published in Electrical Communication.

Glossary of Symbols

I_0 = modified Bessel Function of first kind and zero order
K_0 = modified Bessel Function of second kind and zero order
I_1 = modified Bessel Function of first kind and first order
K_1 = modified Bessel Function of second kind and first order
β = ω/v
k = ω/c
$\gamma^2 = \beta^2 - k^2$
a = radius of the helically conducting cylinder
b = radius of the outer conducting cylinder
v = axial phase velocity
c = velocity of light
ω = radian frequency
ψ = angle made by the helical direction with a circumference
$F(\gamma a)$ = impedance function. See references 1 and 3
$F(\gamma a, \gamma b)$ = impedance function. See reference 3.

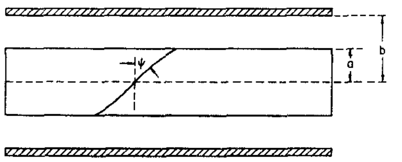

Figure 1 - Model helix. a is the mean radius of the helically conducting cylindrical surface, b is the inner radius of the uniformly conducting, coaxial, cylindrical sheet surrounding the helix, and ψ is the angle defining the direction of conduction along the helix.

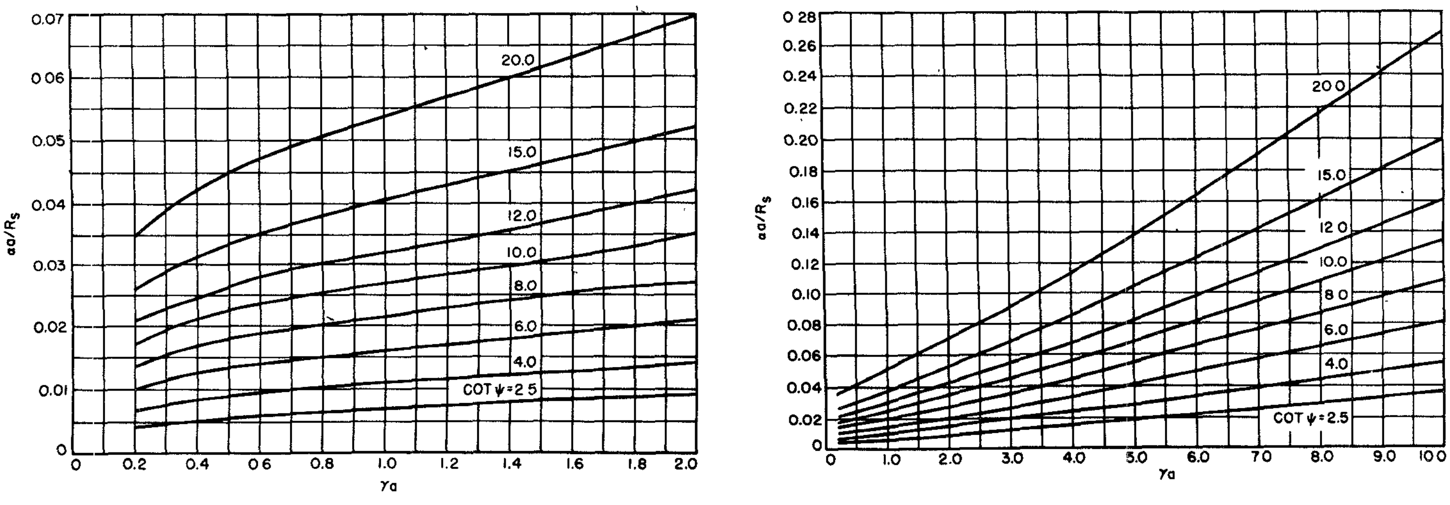

Figure 2 - Helix attenuation due to conductor losses $\alpha a/R_s$, where α = attenuation in nepers per unit length of radius, a = helix mean radius, and $R_s = 2.61 \times 10^{-7} f^{1/2}$ ohms for copper, $4.706 \times 10^{-7} f^{1/2}$ ohms for tungsten, and $4.741 \times 10^{-7} f^{1/2}$ ohms for molybdenum.

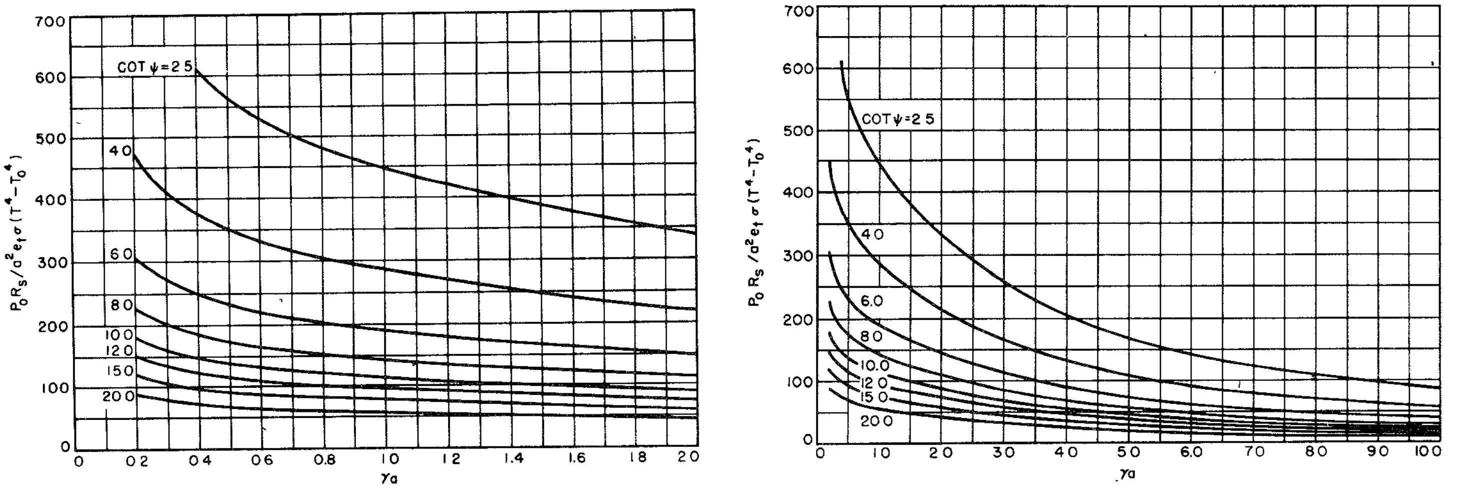

Figure 3 - Power-handling capability. $P_0 R_s / a^2 e_t \sigma (T^4 - T_0^4)$ plotted against γa. P_0 = input power in watts, R_s = skin-effect resistance in ohms, a = helix mean radius in centimeters, e_t = total emissivity, T = helix temperature in Kelvin scale, T_0 = ambient temperature in Kelvin scale, and σ = 5.67×10^{-12} watts per square centimeter per degree^{-4} Kelvin.

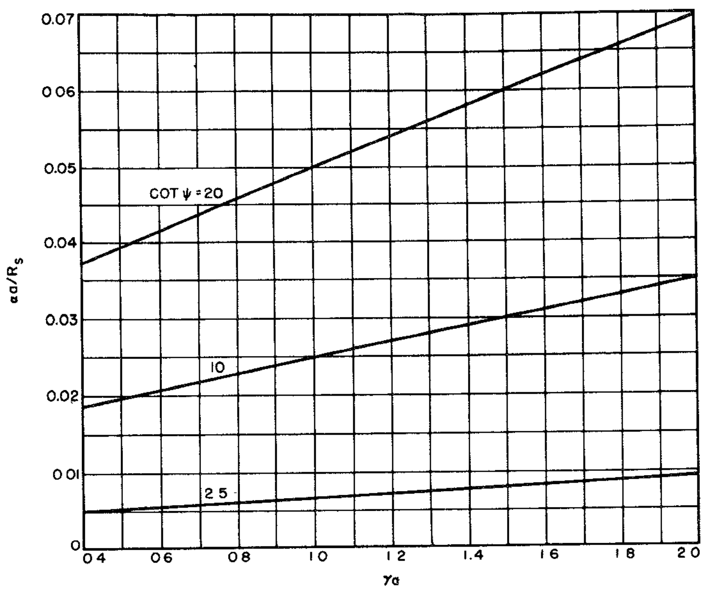

Figure 4 - Helix attenuation due to conductor losses for the case where the helically conducting sheet is surrounded by an outer, uniformly conducting, coaxial cylinder of the same material with a diameter twice that of the sheet and at the same temperature.

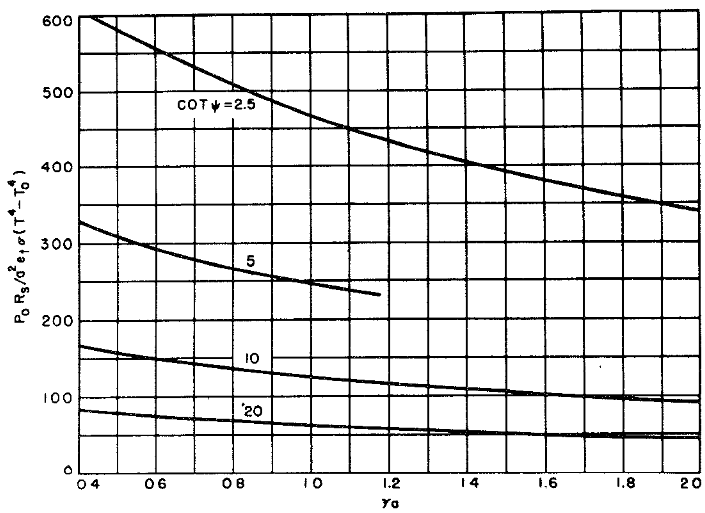

Figure 5 - Power-handling capability as limited by the conductor losses and radiation from the helically conducting sheet.